General Certificate of Secondary Education

Further Mathematics

Unit 1
Pure Mathematics

[GFM11]

Assessment

MARK SCHEME

GCSE MATHEMATICS

General Marking Instructions

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for accurate working, whether in calculation, reading from tables, graphs or answers.

MW indicates marks for combined method and accurate working.

Assessment Objectives
Below are the assessment objectives for GCSE Further Mathematics.

Use and apply standard techniques (AOl)
Candidates should be able to:
- accurately recall facts, terminology and definitions;
- use and interpret notation correctly; and
- accurately carry out routine procedures or set tasks requiring multi-step solutions.

Reason, interpret and communicate mathematically (AO2)
Candidates should be able to:
- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- present arguments and proofs; and
- assess the validity of an argument and critically evaluate a given way of presenting information.

Solve problems within mathematics and in other contexts (AO3)
Candidates should be able to:
- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;
- evaluate methods used and results obtained; and
- evaluate solutions to identify how they may have been affected by assumptions made.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be **followed through** from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

It should be noted that where an error trivialises a question, or changes the nature of the skills being tested, then as a general rule, it would be the case that not more than half the marks for that question or part of that question would be awarded; in some cases the error may be such that no marks would be awarded.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier, only a proportion of the marks will be available (based on the professional judgement of the examiner).

Additional guidance for teachers

These notes explain how the marks allocated in the published mark scheme are to be applied.

In the mark scheme, M indicates method marks and W indicates work marks. MW indicates a combined method and work mark. Work marks should not be awarded if the method is incorrect.

If a candidate misreads a question, eg copies a given equation incorrectly, deduct 1 mark and then FT (follow through), as long as the question is not made easier. However, work marks should not be awarded to answers which are inconsistent with the question, eg negative numbers of people.

If a correct answer in working is transferred incorrectly (or not at all) to the answer line, give BOD (benefit of doubt). However, if the incorrect answer is used in a subsequent section, penalise one mark in the subsequent section and treat the remaining work as a misread.

As a general rule, ignore work that is scored out.

However, if correct working is scored out, but subsequently used correctly, give BOD.

If incorrect working is not scored out but subsequently corrected, give BOD.

If more than one attempt at a question is made and none is scored out, mark the attempt that corresponds to the answer given in the answer line. If no answer is given in answer line, mark the worst attempt, unless the better attempt is clearly the one that is intended to be taken.

If incorrect or unnecessary working is given after a correct answer, ignore this if the correct answer is on the answer line. However, if a subsequent incorrect answer is given on the answer line deduct 1 mark.

Mark answers only in appropriate section. Allow forward marking if carried through into appropriate section, eg if working for part **(ii)** is given in part **(i)** and not repeated but used in part **(ii)**, give appropriate marks in part **(ii)**.

Do not allow backward marking, eg answer to part **(ii)** is given as answer to part **(i)** and no answer, or an incorrect answer, is given in part **(ii)**.

All working must be shown, so answers with no working can get no marks, unless the working is trivial.

Answers should be exact where possible.

Unless specifically stated in the mark scheme, accept one or more dp (decimal places), or 3 significant figures, unless insufficient accuracy leads to very inaccurate or meaningless answer.

Ignore slight rounding errors, eg arising from calculations using values to 2 rather than 3 or more places. Accept 1.5 instead of 1.50 for an answer required to 2 dp. Ignore truncation error in the last digit unless a specific accuracy is required, eg 0.345 truncated to 0.34.

1 MW1 MW1 MW1 – One mark for each correct term.
For the second term accept $4x^{-4}$ or $4/x^4$. Accept 12/3 instead of 4.
Do not accept a double negative, eg $-(-4)x^{-4}$.

2 MW1 MW1 MW1 MW1 – One mark for each correct term.
For the second term accept x^{-1} or $1/x$.

3 **(i)** MW1 MW1 – MW1 for each angle. Accept answers correct to 1 dp or more.
Deduct 1 mark for one or more additional angles.

No marks in part (i) if answers are in radians.

(ii) M1 – for an equation using at least one angle from **(i)**.
Accept any angle, even an incorrect angle, from **(i)**, as long as it is in the range $-180° \le x \le 180°$.

W1 W1 – W1 for each angle.
Allow FT from angles in **(i)**, as long as the angles are in the correct ranges in both parts **(i)** and **(ii)**.

In part (ii) award the M1 mark at most, ie no work marks, if angles are in radians.

4 **(i)** M1 – for the use of Pythagoras, which must be clearly shown
May be stated as eg $(2-x)^2 = 9^2 - 4^2$ (or 81 – 16),
but $(2-x)^2$ must be stated explicitely.

W1 – for second or third line of solution correctly shown.
Do not award if only the first line and last line are shown.
Do not award if M1 has not been given.

(ii) No marks in this section if completing the square method is not explicitely shown.

M1 W1 – M1 for $(x-2)^2$ or $(2-x)^2$
Any other expression squared gets no marks in this section.
Allow no FT from an incorrect equation in **(i)** as the correct equation is given.

W1 for correctly subtracting 4 in a correct expression.
Accept –65 instead of –4 – 61 in initial line.

W1 – for solving for x.

M1 – for giving answer as $2 - \sqrt{65}$.
If first M1 W1 awarded and $2 \pm \sqrt{65}$ given as answer, give 3 marks out of 4.
If first M1 W1 awarded and only $2 - \sqrt{65}$ given as answer, give 4 marks out of 4.
If first M1 W1 awarded and only $2 + \sqrt{65}$ given as answer, give 2 marks out of 4.

5 M1 – for writing a correct quadratic inequality.

MW1 – for finding the roots of the quadratic.

M1 – for method for finding the appropriate areas for the answer.
Award this mark for any valid method, eg sketch or inserting appropriate values into the quadratic, eg $x = -2$, $x = 0$ and $x = 2$.

W1 W1 – one mark for each region stated correctly.

If > 0 is left out in the initial inequality but the rest of the work is correct and the final answer is correct, award 5 marks.
If they start with $2x^2 - x - 3 = 0$ and the final answer is correct, award 4 marks.
If the method is correct and the final answer is $-1 < x < 3/2$, award 3 marks.
If the method is correct and the final answer is $x \leq -1$ or $x \geq 3/2$, award 4 marks (ie do not penalise the same mistake twice).

6 **(i)** MW1 – for matrix multiplication.
If this is not shown, award the mark if $2x = -6$ is shown.

W1 – for a correct answer.
If the correct answer is given with no working, award the full 2 marks.

(ii) MW1 – for an equation correctly worked out from the determinant.
Allow FT from an incorrect value of x in **(i)**.

W1 – for a correct value of 8.
Do not award this mark if x is incorrect in **(i)**, even if 8 is obtained.

7 **(a)** M1 – for taking logs of both sides.

M1 – for correctly dealing with indices.
If brackets are missing, eg $3x - 2 \log 4$, give BOD for this mark if subsequent work treats the expression as if brackets were there.

W1 – for correctly gathering terms in x to one side.

M1 – for a correct method to find x.
Allow FT from an incorrect expression in the previous line.

W1 – for a correct answer.

Correct values for log 4 (0.602060) and log 6 (0.778151) can be used throughout.

Example: $(3x - 2) \log 4 = (x - 1) \log 6$
$3x \log 4 + x \log 6 = 2 \log 4 - \log 6$
$x = (2 \log 4 - \log 6) / (3 \log 4 + \log 6)$
gets M1, M1, W0, M1, W0.

(b) M1 – for $y = 2 \log 2$.

MW1 – for $y = -2z$.
The expression $z = -\log 2$ need not be shown.

8 **(i)** MW1 MW1 M1 MW1 – One MW1 mark for each of the three correctly factorised expressions.
The M1 mark is for correctly changing the division to a multiplication.
Award this mark even if no attempt, or a wrong attempt, is made to factorise.

W1 – for a correct answer.

(ii) W1 – for a correct factorised expression.

M1 W1 – M1 for using the correct lowest common denominator.
W1 for a correct numerator.

MW1 – for correctly factorising the numerator.
Allow no FT from an incorrect quadratic expression, as it would be unlikely to give similar factors.

W1 – for correct answer.
Allow no FT as need to see cancellation of terms to give correct answer.

9 **(i)** MW1 – for factorising quadratic equation.
Accept use of quadratic formula method.

W1 – for correct coordinates.
Must be expressed as coordinates; $x = -1$ or 4 is not sufficient.

(ii) M1 W1 – M1 for clear attempt at differentiation.

W1 for differentiating correctly.

M1 – for putting $\mathrm{d}y/\mathrm{d}x = 0$ and getting the correct solution.
If $\mathrm{d}y/\mathrm{d}x$ is incorrect, allow FT for x as long as expression for $\mathrm{d}y/\mathrm{d}x$ is linear.

W1 – for getting the turning point.
Allow FT from an incorrect value of x, as long as work is of similar standard.

(iii) MW1 – for showing turning point is a maximum.
Stating $\mathrm{d}^2y/\mathrm{d}x^2 = -2$ or $\mathrm{d}^2y/\mathrm{d}x^2 < 0$ is sufficient.
Showing $\mathrm{d}y/\mathrm{d}x$ goes from positive to zero to negative is acceptable.

(iv) M1 W1 – M1 for a correctly shaped quadratic curve.

W1 for intercepts on x-axis and turning point clearly marked.
The x values must be correct, but allow FT from the y value from **(ii)**.

(v) M1 – for knowing to integrate between –1 and 4.
Allow FT from incorrect x values from **(i)**.

MW1 – for all terms correct in the integration.
No penalty for a constant of integration included.

M1 – for putting in their values of the limits correctly to evaluate the integral.

W1 – for the correct answer only.
Thus if $x = -1$ or 4 are incorrect, it is still possible to get M1, MW1, M1, W0.

10 MW1 – for correctly finding **B** + **C**.

MW1 MW1 – These two MW1 marks are independent of each other.

MW1 for 1/2.
Only give this mark if they have multiplied by a matrix, even if the matrix is not correct.

MW1 for a correct matrix.
Only give this mark if they have multiplied by a fraction, even if the fraction is not correct.

M1 – for multiplying the matrices in the correct order.
Allow FT for $\mathbf{A}^{-1}$, but they must have been awarded at least 1 mark for their $\mathbf{A}^{-1}$.
Allow FT for **B** + **C**.

W1 – for the correct answer only.

11 **(i)** W1 – for a correct equation.

(ii) M1 – for a correct equation from the information given which leads to the given equation.
M1 – for a correct equation showing ⅔, $(y + 25)$ and ½.

(iii) **No marks in this section if no method is shown.**

M1 W1 – M1 for eliminating a variable to get an equation in 2 variables.

W1 for a correct equation.

M1 W1 – M1 for a second equation in 2 variables.
Must be the **same** 2 variables as the previous equation.

W1 for a correct equation.
Must have obtained the M1 for the second equation.

M1 – for eliminating a variable to get an equation in 1 variable.

W1 – for a correct solution.
Allow FT from an incorrect equation, as long as the answer is a positive integer below 180.

M1 – for using back substitution to get the remaining 2 variables.
Must get answers for both variables.

W1 – for answer.
Allow FT, as long as all passenger numbers are positive integers which total 180.

(iv) MW1 – for getting answers.
Allow FT, as long as all passenger numbers are positive integers which total 180.

W1 – for answer.
Allow FT, as long the previous MW1 has been obtained.

12 **(i)** MW1 – for correct expression for derivative.

MW1 – for gradient of tangent at $x = 1$.
Allow FT from incorrect expression for derivative, as long as it is a quadratic expression with 2 terms.

MW1 – for equation of tangent.
Allow FT from incorrect gradient.

(ii) MW1 – for gradient of normal.
Allow FT from incorrect expression for derivative, as long as the gradient of the normal is given as $-1/(\mathrm{d}y/\mathrm{d}x)$.

MW1 – for equation of normal.
Allow FT from incorrect gradient of normal, as long as it was calculated from $-1/(\mathrm{d}y/\mathrm{d}x)$.

(iii) M1 – for setting the equations from parts **(i)** and **(ii)** equal. Equations need not necessarily be correct.

W1 – for answer.
Exact answer must be given, eg no rounding to 2 dp.
Allow FT as long as equations lead to an exact answer which cannot be expressed exactly to any number of decimal places.

13 (i) M1 – for expression for log P.
This expression must appear somewhere in part **(i)**.

W1 W1 – W1 for log R values correct to 3 dp.
W1 for log P values correct to 3 dp.
Allow 1 value in each column to be out by 1 digit in the third dp without penalty.
If values are given to 2 dp, give 1 mark if all values in both columns are correct to 2 dp.
If values are given to more than 3 dp, give 1 mark if all values would be correct to 3 dp.

W1 – for labels on axes.
Labels must be logs.
Accept names instead of R or P, eg log (population).

If log *R* is on the *x*-axis, lose this mark and the following two marks.

W1 – for points plotted correctly.
Allow FT from table and accept points which are plotted slightly off.

W1 – for straight line, plotted reasonably well through all points.
No FT from incorrect points, except where points had a slight rounding error, or were plotted slightly off.

(ii) M1 – for calculating the gradient.
Must have difference in log P values divided by difference in log R values, even if log P and log R are on the wrong axes.
Values may be from the table or read from the graph.

W1 – for value of gradient.
Accept 2 or more dp.
Accept small variations due to rounding from using different values in the calculation.

M1 – for method for calculating a.
Using logs is acceptable, but final answer must be given for a and not log a.

W1 – for value for a.
Answer must be given as an integer.
Accept small variations due to rounding from using different values in the calculation.

(iii) MW1 – for answer.
Allow FT from values for a and n, but method must be correct and answer must be in the range 100,000 to 150,000.

14 **(i)** M1 – for correct expression for perimeter. Need not be simplified.

W1 – for correct expression for y. No FT from incorrect expression for perimeter.

(ii) MW1 – for expression for A. Allow FT from incorrect expression for y in **(i)**, as long as final expression is quadratic.

(iii) M1 W1 – M1 for differentiating expression for A and setting equation equal to zero.

W1 for a correct expression for the derivative. No FT if an incorrect expression from **(ii)** is used.

W1 – for correct values for x and y.

MW1 – for justifying the maximum. Stating second derivative is –2 or < 0 is sufficient. Allow FT from incorrect expression for derivative, as long as it proves a maximum. Showing first derivative goes from positive to zero to negative is acceptable.

		AVAILABLE MARKS
1 $y = 6x^3 - \frac{4}{3x^3} + 2x$		
$= 6x^3 - \frac{4}{3}x^{-3} + 2x$		
$\frac{dy}{dx} = 18x^2 + 4x^{-4} + 2$	MW1 MW1 MW1	
or $18x^2 + \frac{4}{x^4} + 2$		3
2 $\int \left(\frac{x^3}{4} - \frac{1}{x^2} + 5\right) dx$		
$= \int \left(\frac{x^3}{4} - x^{-2} + 5\right) dx$		
$= \frac{1}{16}x^4 + x^{-1} + 5x + c$	MW1 MW1 MW1 MW1	
or $\frac{1}{16}x^4 + \frac{1}{x} + 5x + c$		4
3 **(i)** $\tan x = 1.5$		
$x = 56.310°$ or $-123.690°$		
$x = 56.31°$ or $-123.69°$	MW1 MW1	
(ii) $\tan(3\theta - 10°) = 1.5$		
$3\theta - 10° = 56.310°$ or $-123.690°$	M1	
$3\theta = 66.310°$ or $-113.690°$		
$\theta = 22.10°$ or $-37.90°$	W1 W1	5

		AVAILABLE MARKS

4 **(i)** By Pythagoras

$$(2-x)^2 + 4^2 = 9^2 \qquad \text{M1}$$

$$x^2 - 4x + 4 + 16 = 81 \qquad \text{W1}$$

$$x^2 - 4x + 20 = 81$$

$$x^2 - 4x - 61 = 0$$

(ii) $(x-2)^2 - 4 - 61 = 0$ M1 W1

$$(x-2)^2 - 65 = 0$$

$$(x-2)^2 = 65$$

$$x - 2 = \pm\sqrt{65}$$

$$x = 2 \pm \sqrt{65} \qquad \text{W1}$$

But the side length $(2-x)$ cannot be negative

$$\therefore\ x = 2 - \sqrt{65} \qquad \text{M1} \qquad 6$$

5 $x(2x - 1) - 3 > 0$

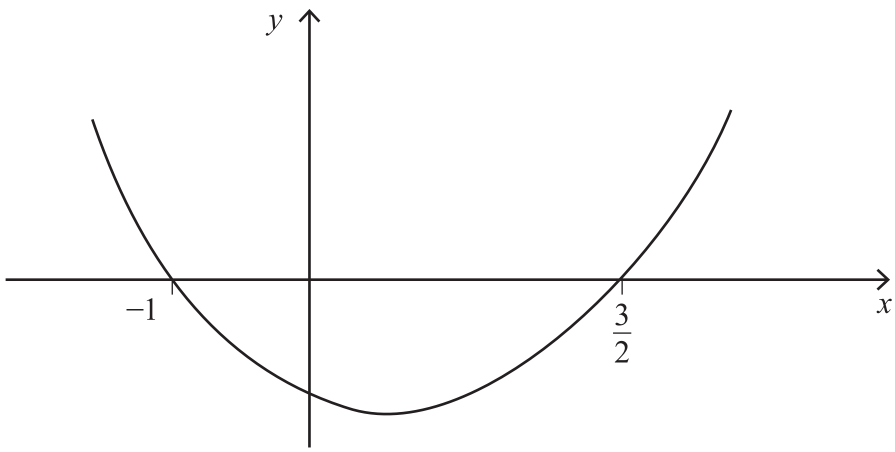
$$2x^2 - x - 3 > 0 \qquad \text{M1}$$

Find roots

$(2x - 3)(x + 1) = 0$

$x = \frac{3}{2}$ or $x = -1$ MW1

Sketch



M1

$2x^2 - x - 3 > 0$ when $x < -1$ or $x > \frac{3}{2}$ W1 W1 5

6 **(i)** $\mathbf{P\,Q} = \mathbf{R}$

AVAILABLE MARKS

$$\begin{bmatrix} x & -4 \\ 6 & y \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -6 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 2x \\ 12 \end{bmatrix} = \begin{bmatrix} -6 \\ 12 \end{bmatrix}$$ MW1

$$2x = -6$$

$$x = -3$$ W1

(ii) $$\det \mathbf{P} = \det \begin{bmatrix} -3 & -4 \\ 6 & y \end{bmatrix}$$

$$= -3y - (-24) = -3y + 24$$

$$-3y + 24 = 0$$ MW1

$$y = 8$$ W1 4

AVAILABLE MARKS

7 (a)

$4^{3x-2} = 6^{x-1}$		
$\log 4^{3x-2} = \log 6^{x-1}$	M1	
$(3x-2)\log 4 = (x-1)\log 6$	M1	
$3x\log 4 - 2\log 4 = x\log 6 - \log 6$		
$3x\log 4 - x\log 6 = 2\log 4 - \log 6$	W1	
$x(3\log 4 - \log 6) = 2\log 4 - \log 6$		
$x = \dfrac{2\log 4 - \log 6}{3\log 4 - \log 6}$	M1	
$x = 0.41$	W1	

(b)

$y = \log 4$		
$y = \log 2^2$		
$y = 2\log 2$	M1	
$z = -\log 2$		
$y = -2z$	MW1	7

		AVAILABLE MARKS
8 **(i)** $\dfrac{x^2 - 5x + 6}{x^2 - 9x + 18} \div \dfrac{x - 2}{2x^2 - 12x}$		
$= \dfrac{(x-3)(x-2)}{(x-6)(x-3)} \times \dfrac{2x(x-6)}{x-2}$	MW1 MW1 M1 MW1	
$= 2x$	W1	
(ii) $\dfrac{2x}{x-1} - \dfrac{x+1}{x^2 - x}$		
$= \dfrac{2x}{x-1} - \dfrac{x+1}{x(x-1)}$	W1	
$= \dfrac{2x^2 - (x+1)}{x(x-1)}$	M1 W1	
$= \dfrac{2x^2 - x - 1}{x(x-1)}$		
$= \dfrac{(2x + 1)(x-1)}{x(x-1)}$	MW1	
$= \dfrac{2x + 1}{x}$	W1	10

AVAILABLE MARKS

9 **(i)** $y = -x^2 + 3x + 4 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$ MW1

$x = -1 \quad \text{or} \quad x = 4$

Crosses x-axis at $(-1, 0)$ and $(4, 0)$ W1

(ii) $\dfrac{dy}{dx} = -2x + 3$ M1 W1

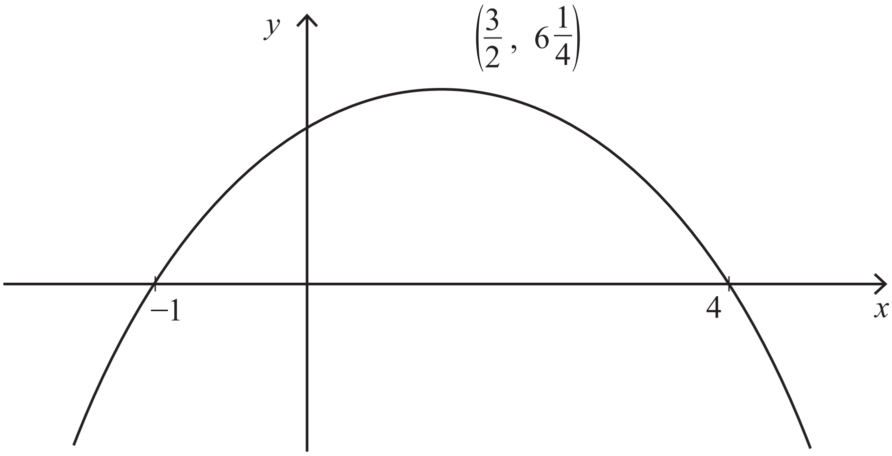
$= 0 \text{ when } x = \dfrac{3}{2}$ M1

$y = -\left(\dfrac{3}{2}\right)^2 + 3\left(\dfrac{3}{2}\right) + 4 = 6\dfrac{1}{4}$

Turning point at $\left(\dfrac{3}{2},\ 6\dfrac{1}{4}\right)$ W1

(iii) $\dfrac{d^2y}{dx^2} = -2 < 0 \therefore$ maximum MW1

(iv)



M1 W1

AVAILABLE MARKS

(v) $A = \int_{-1}^{4} (-x^2 + 3x + 4)\,dx$ M1

$= \left[-\frac{x^3}{3} + \frac{3}{2}x^2 + 4x\right]_{-1}^{4}$ MW1

$= \left[-\frac{64}{3} + 24 + 16\right] - \left[\frac{1}{3} + \frac{3}{2} - 4\right]$ M1

$= 20\frac{5}{6}$ or 20.83 W1 13

10 $\mathbf{AX} = \mathbf{B} + \mathbf{C} = \begin{bmatrix} -2 \\ 7 \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$ MW1

$\mathbf{X} = \mathbf{A}^{-1} \begin{bmatrix} -3 \\ 5 \end{bmatrix}$

$\mathbf{A}^{-1} = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}^{-1} = \frac{1}{2}\begin{bmatrix} -5 & 3 \\ -4 & 2 \end{bmatrix}$ MW1 MW1

$\mathbf{X} = \frac{1}{2}\begin{bmatrix} -5 & 3 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 5 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 30 \\ 22 \end{bmatrix}$ M1

$\mathbf{X} = \begin{bmatrix} 15 \\ 11 \end{bmatrix}$ W1 5

11	**(i)** $70x + 55y + 35z = 9325$	W1	**AVAILABLE MARKS**
	Divide by 5		
	$14x + 11y + 7z = 1865$		
	(ii) $\frac{2}{3}x + (y + 25) + \frac{1}{2}z = 180$	M1 M1	
	$\frac{2}{3}x + y + \frac{1}{2}z = 155$		
	$4x + 6y + 3z = 930$		
	(iii) $x + y + z = 180$ [1]		
	$14x + 11y + 7z = 1865$ [2]		
	$4x + 6y + 3z = 930$ [3]		
	[2] − [1] × 7 $\rightarrow$ $7x + 4y = 605$ [4]	M1 W1	
	[3] − [1] × 3 $\rightarrow$ $x + 3y = 390$ [5]	M1 W1	
	[5] × 7 − [4] $\rightarrow$ $17y = 2125$	M1	
	$y = 125$	W1	
	From [5] $x = 390 - 3(125) = 15$		
	From [1] $z = 180 - 15 - 125 = 40$	M1	
	Premium = 15, Economy = 125, Stand-by = 40	W1	
	(iv) For second flight		
	Premium $= \frac{2}{3}(15) = 10$		
	Economy $= 125 + 25 = 150$		
	Stand-by $= \frac{1}{2}(40) = 20$	MW1	
	Total income $= (10 \times 70) + (150 \times 55) + (20 \times 35)$		
	$= £9650$	W1	13

			AVAILABLE MARKS
12	**(i)**	$y = \frac{2}{3}x^2(x-3) = \frac{2}{3}x^3 - 2x^2$	
		$\frac{dy}{dx} = 2x^2 - 4x$ MW1	
		For the **tangent** $x = 1, \; m = 2-4 = -2$ MW1	
		Equation of tangent $y = -2x + c$ At $\left(1, -1\frac{1}{3}\right) \quad -1\frac{1}{3} = -2 + c$ $c = \frac{2}{3}$ $y = -2x + \frac{2}{3}$ MW1	
	(ii)	For the **normal** $x = -1, \; \frac{dy}{dx} = 2 + 4 = 6$	
		Gradient of normal $= -\frac{1}{6}$ MW1	
		Equation of normal $y = -\frac{1}{6}x + c$ At $\left(-1, -2\frac{2}{3}\right) \quad -2\frac{2}{3} = \frac{1}{6} + c$ $c = -2\frac{5}{6}$ $y = -\frac{1}{6}x - 2\frac{5}{6}$ MW1	
	(iii)	At the point of intersection $-\frac{1}{6}x - 2\frac{5}{6} = -2x + \frac{2}{3}$ M1	
		$-x - 17 = -12x + 4$ $11x = 21$ $x = \frac{21}{11}$ or $1\frac{10}{11}$ W1	7

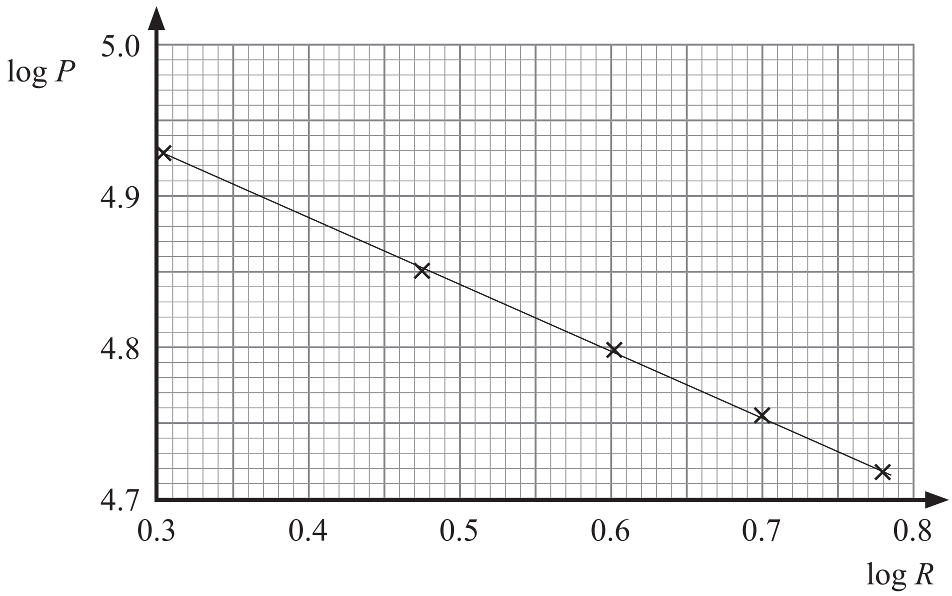
13 (i) $P = aR^n$

$\log P = \log aR^n = \log a + \log R^n$

$\log P = n \log R + \log a$ M1

$\log R$	$\log P$
0.301	4.929
0.477	4.851
0.602	4.799
0.699	4.756
0.778	4.716

W1 W1



W1 (labels)

W1 (points)

W1 (line)

(ii) $n = \dfrac{4.716 - 4.929}{0.778 - 0.301} = -0.44654 \to -0.45$ M1 W1

$P = aR^n$

$85\,000 = a2^{-0.44654}$

$a = 115\,835$ M1 W1

AVAILABLE MARKS

(iii) For Belfast, $R = 1$

$P = 115\,835\,(1)^{-0.45} = 115\,835$ MW1

This is not approximately 333 000, so the formula does not hold. 11

Alternative solution

$333\,000 = 115\,835\,R^{-0.45}$

$$R = \left(\frac{330\,000}{115\,835}\right)^{-\frac{1}{0.45}}$$

$= 0.0957$ MW1

This is not approximately 1, so the formula does not hold.

14 **(i)** Perimeter fence $= x + (x-4) + y + y$ M1

$= 2x + 2y - 4$

$= 100$

$2y = 104 - 2x$

$y = 52 - x$ W1

(ii) $A = xy = x(52-x)$

$= 52x - x^2$ MW1

(iii) $\dfrac{dA}{dx} = 52 - 2x = 0$ M1 W1

$x = 26$

$y = 52 - 26 = 26$ W1

$\dfrac{d^2A}{dx^2} = -2 < 0 \;\therefore$ maximum MW1

Length $= 26$ m, width $= 26$ m 7

Total **100**